Book Review

Duel at Dawn: Heroes, Martyrs and the Rise of Modern Mathematics, by Amir Alexander. Harvard University Press, 2010. ISBN 978-0-674-04661-0. $19.11 on amazon.com—**Reviewed by N.W.J. Hazelton**

Dawn, May 30, 1832. Two young men meet at Gentilly on the outskirts of Paris to fight a duel. At issue is the honor of a young woman. Both men are members of revolutionary groups and have been active in the political upheavals of the era. They have agreed that only one of the pistols will be loaded, leaving the affair to chance. The face each other, and at the signal a shot rings out. One falls, mortally wounded, and is taken to the nearby Cochin hospital. The next morning, Évariste Galois dies in the arms of his brother, Alfred. He is just 20 years old.

A tragic death in the style of the Romantic era, then at its height, but Galois is not a Romantic poet or artist. A revolutionary, a radical who has spent a year in prison for his politics, a young man intent on stirring up his staid university, a firebrand who sees much of the world in terms of black and white, truth and falsehood: Galois is all of these. But he is also a brilliant mathematician, seen as being of great promise by the leading scientists of the day, and has already played a significant role in changing the fundamental nature of mathematics.

The story of Galois has become a legend of modern mathematics. He has been changed into a Romantic martyr for truth and modern mathematics, his nature revised from self-destructive to self-sacrificing. His life and death marks a turning point in the ethos of mathematics. Amir Alexander's book explores the nature of this turning point and the mindset that developed in mathematics from this period onward.

Most of the mathematicians of the 18th century, such as Euler, d'Alembert and Lagrange, were men of the Enlightenment. Many of them were also engineers, and Alexander frequently calls them geometers—'Earth measurers' or 'surveyors' in French. They believed that mathematics existed to illuminate Nature, to provide order and insight to help understand the chaos of the unexplained world. Mathematics was therefore intimately and intrinsically connected to reality, and the success of mathematics was founded on its usefulness. Proof was therefore less important than utility, and geometry held the central place, just as it had since the Ancient Greek philosophers.

The 'new' mathematicians of the Romantic era took a different view. Mathematics should be a self-contained system, divorced from reality, with proof being the key issue and only path to truth. Algebra became central, and mathematics was to follow the Euclidean model of carefully building each new component by proof from earlier proved theorems. Many mathematicians of the day took off in new directions, exploring theoretical areas that appeared to have no connection to reality. Bolyai explored non-Euclidean spaces. Galois and Abel developed critical aspects of group theory. Riemann moved into extremely abstract algebraic representations of spaces. Other mathematicians, such as Cauchy, re-developed earlier work, such as calculus, in terms of the requirement for rigorous proof.

Alexander explores these changes by exploring the stories of the individual mathematicians and their interactions over time. What emerges from this exploration is the origins of the sharp divide between 'pure' and 'applied' mathematics, a curse of modern times, which Alexander does not really explore. He also fails to explore the sneering contempt that many pure mathematicians seem to feel and express for applied mathematics.

Since C.P. Snow discussed the divide between 'pure' and 'applied' science in the UK in the 1950s, this divide in science has tended to be bridged over from both sides. Physicists and astronomers, who occupy areas of science that are less 'applied,' nonetheless are focused on explanation and understanding of the natural world. But the divide is still alive and strong in mathematics.

This peculiar aspect of modern mathematics is illustrated in its legends and 'mythical' figures. Alexander discusses several, such as G.H. Hardy, Srinivasa Ramanujan, Kurt Gödel, John Nash, and Grigory Perelman in more recent times. The Romantic nature of these figures, their struggle to overcome the opposition and barriers to their genius, their 'vows of poverty,' and their dedication to the truth, beauty and purity of mathematics, illuminates the ethos of modern mathematics.

A central tenet of mathematics today is the need to forsake the world, dedicate oneself to the purity and beauty of the discipline, and struggle against an uncomprehending and unenlightened world. Mathematics is painted in almost religious or poetic terms. How John von Neumann and Roger Penrose would fit into this picture, Alexander never explains.

There are many lines of discussion that can come from such a picture and an important one is: How do you recruit for such a discipline?

The solution appears to be to appeal to those people who have an interest in abstract mathematics, and focus the efforts of mathematicians on the important goal of perpetuating the discipline. Along the way, the disconnection from the rest of the world is achieved. Those who fall short of the required level of dedication, self-sacri-

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fice, and true appreciation of truth and beauty are left by the wayside. They are the failures, the unworthy, the infidels, the barbarians at the gates. And they make up most of the world.

Of course, this is an exaggeration, but there is a kernel of truth here. This concept is a thread in the ethos of modern mathematics, something worked into the soul of modern mathematicians. It has a major impact on how mathematics is taught, and therefore on how well it is integrated into modern society and its thinking. By focusing on the purity of mathematics, mathematicians have alienated their craft from much of society, thereby ensuring their continued isolation, in keeping with those Romantic ideals.

But alienation from the rest of society means that mathematical literacy in the general population is low, most people hate and fear math, and we have the foundations of an anti-mathematical society, complete with antisocial mathematicians. This is not a good thing today, when we have never had greater need of mathematical literacy.

The geomatics profession is very much a discipline of applied mathematics. Geometry is at the core of what we do, and we are very much connected to the real world, as were almost all mathematicians from the Ancient Babylonians to the Enlightenment geometers. But mathematics is a stumbling block to recruiting for us, and it's the same in engineering. Mathematics as a discipline tends to drive away mere practical people, and that makes it hard for us to recruit. It may be timely for the applied mathematical disciplines to take back applied mathematics, to take charge of the foundations of our own disciplines. This is a complex issue, but we need to make sure that mathematics serves us, not the other way around.

Alexander's book raises some interesting points, but it is short on analysis of the consequences of the change in the fundamental nature of mathematics that it chronicles. It is also a bit repetitive: I don't know how many times I read that Galois was built up to be a Romantic hero of mathematics, and how many times the same points were made about Abel. Perhaps Alexander should have taken greater heed of the notion of 'proof' in his telling of the story. I would have preferred less time spent repeating these ideas and more looking at the consequences of these changes. That said, it is an interesting interpretation of history, which provides some useful insights into the group psychology of mathematics and mathematicians today. ■